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OPTIMAL MULTI-ECHELON STOCKAGE POLICIES WITH CONSTRAINTS

W. Karl Kruse

Army Logistics Management Center
Fort Lee, Virginia

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ABSTRACT

An S,S-1 multi-echelon model is used to develop optimal stockage policies for a catalog of items. The items were the repair parts of a rough terrain forklift on which a military essentiality methodology had been tested. The model included such factors as holding cost, performance objective, military essentiality, and transportation cost. An optimization algorithm was developed and a constraint manipulation technique experimented with.

SUMMARY

1. Background

The Army retail logistics system is a dynamic, diverse, and complex system which has direct contact with the Army combat units. As such, its proper functioning has direct bearing on the National Defense. In the past, retail logistics expenditures have been wasteful. Lack of understanding of the system along with the awareness of its importance have encouraged over-spending. A particular area where over-spending often occurs is in the stockage of repair parts for maintenance actions at direct support units, general support units, and field depots. This paper presents a model of the retail system which enables development of a multi-echelon stockage policy that tells where and in what quantities parts should be stocked to meet a given performance measure. The policies are sub-optimal in that a restriction is placed on the form of the policy. Nevertheless, the model provides insight into and understanding of the system and can be used as an aid in developing more realistic stockage policies.

2. Objectives and Scope

The objectives of the study were -

a. Develop a feasible optimization algorithm suitable for developing multi-echelon stockage policies on a large catalog of items.

b. Develop a technique to introduce the system constraints in the model and determine a method to solve for optimal policies under constraints.

c. Illustrate how military essentiality measures can be used in stockage decisions.

d. Illustrate how transportation and inventory cost can be jointly considered in stockage decisions.

A multi-echelon model originally developed by IRO for the Brown Board has been extended to include military essentiality, stockage constraints, and transportation costs. This model is based on a queueing theorem of Palm which relates the amount of stock at a unit to its average resupply time when the demand on that unit is Poisson. Imposing an S, S-1 policy on each unit insures Poisson demand at all units. The stockage policies of the upper levels are related to the supply times of the lower levels and Palm's Theorem is applied to give the probability distribution of inventory.

A search type algorithm was developed to solve for the single item optimum stockage policy. It was designed to be fast so as to be feasible for use on a large catalog of items.

The model was then extended to include the stockage constraints by use of the Generalized Lagrange Multiplier method. An algorithm was developed to solve the constraint model.

3. Conclusions and Findings

a. Optimization of the unconstrained model can be accomplished economically on a digital computer and the optimum policies for a large catalog of items can be found quickly.

b. Optimization of the constraint model is substantially more difficult. Only partial success can be reported with the algorithm developed during this study; further work is required before a thorough evaluation of the algorithm can be made.

c. A logical method of employing military essentiality measures in the model was found. An essentiality measure suitable for use in this model can be found by relating target availabilities to essentiality classes.

d. Transportation costs are a substantial portion of total stockage costs. Choosing stock levels and transport mode jointly results in substantial savings.

4. Future Work

IRO intends to continue its research into multi-echelon models. Additional investigation will be made into solution of the constraint problem. New areas to be investigated include batch ordering, i.e., R,Q type policies and cross levelling between units.

CHAPTER I

1.1 Introduction

When providing repair parts for maintenance, a logistics system must procure the parts and distribute them to the user. For the purposes of this paper it is possible to decompose the logistics system into a wholesale system and a retail system on the basis of the function of procurement and distribution. The NICPs are said to make up the wholesale system because they are the procurement agents. The logistics functions below the NICPs are designed primarily for efficient distribution of the procured parts to the users. The units which provide these functions are said to compose the retail logistics system.

Within the past decade or so, the Army has devoted considerable effort to the study of its wholesale operations. Much has come from the studies and today the Army is on the way to developing truly integrated wholesale logistics. There is still much which needs to be done in the study of the retail system. Off hand, it might be expected that the retail system would have commanded the most study effort, since the Army has more resources invested in the retail system than in the wholesale. In terms of possible economies, the efficient operation of the retail system is of greater benefit than the efficient operation of the wholesale. However, the same reason that makes the retail system so important economically, has hindered the development of integrated retail supply policies; for while the diversity and complexity of the retail system results in enormous resource expenditures, it also makes analysis extremely difficult.

This paper represents a model of the retail system which makes it possible to evaluate the effect of stockage anywhere in the system on supply performance. By limiting the scope of the analysis a tractable model has been constructed.

1.2 Retail Logistics System

1.2.1 Network Structure

The retail logistics system can conveniently be portrayed as a network. Fundamentally, a network representation is merely a set of points called nodes connected by a set of lines called branches. The network shown below depicts a three echelon system.

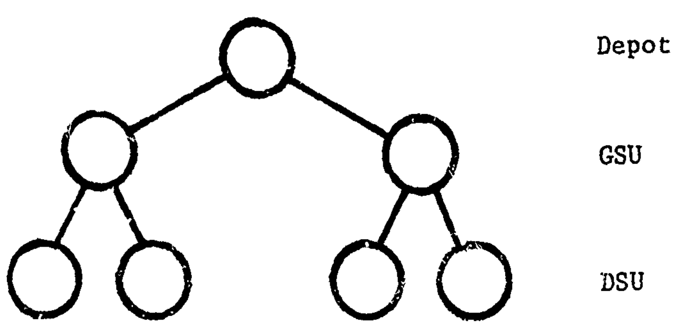


Figure 1

Each node in the network represents a stockage point and the branches indicate the paths requisitions for parts are permitted to take. In other words, the branches indicate the relationships between nodes. Thus, in the network of Figure 1, stockage points in level one may requisition parts from only a given point in level two, and points in level two can order only from level three.

While it is possible to conceive of any number of branch-node interconnections, the Army retail system is portrayed by an arborescent network such as that in Figure 1. That is, the network has a tree-like structure in which each point branches into several points in a lower echelon.

The levels of the system can easily be identified by numerical indexes, but for the purposes of this paper the identification will be specialized by the use of Army terminology. However, keep in mind that the use of Army terminology is for the purpose of discussion and the reader should not lose sight of the generality of the basic model. We will be concerned with a three echelon system. The lowest level shall be called the direct support level and, of course, will be composed of direct support units. Direct support units are close to the organizational units and must be able to provide responsive replenishment and maintenance support. Furthermore, they are required to be mobile so they can remain in close contact with the field at all times. Immediately above the direct support level is the general support level. General support units provide support of a more complex nature than direct support, but they are not as responsive and they do not have as great a mobility. The highest level in the structure is the theater depot. Mobility is not required of the depot but it must perform the most complex support activities in the theater.

1.2.2 Requisitioning Procedures

The requisitioning procedures to be discussed are for routine requisitions. Accepting Figure 1 as representative of a typical system, it can be seen that each unit in a given level has only one unit in the next higher level to which it goes for re-supply. Thus, if a unit wishes to obtain an item, it requisitions that item from its supplier in the next echelon. The item is issued to the requisitioner if it is available. If it is not available, the requisition is backordered and later filled on a first come first served basis. For example, when a DSU requires an item it places an order for that item with the GSU supporting it. In the case that the GSU does not stock the item, a requisition is forwarded to the depot in behalf of the DSU. Again, the depot may or may not stock the item. If it does stock, it is sent to the DSU as soon as possible. If it does not stock the item, a requisition is forwarded to the rear supply which then fills the DSU's order.

1.2.3 Re-Supply Times

One of the most important characteristics of the retail system in so far as stockage is concerned is the re-supply times existing between

the echelons. The re-supply time is composed of a shipping and processing time and a waiting time for the issueable item. The wait is either zero or is some finite time if no parts are available for issue. Clearly, then the re-supply time that a unit experiences depends upon the availability of the item at the level above. The re-supply times identified in table below are the times corresponding to the shipping and processing of the order and do not include the wait portion. Since the processing time a requisition receives at a unit depends upon whether or not the item is stocked at that unit, the resupply times must be identified for all possible stockage conditions. Sdsu, Sgsu, and Sdepot stand for the stock levels at DSU's, GSU's, and depots respectively. A * indicates some positive stockage.

Table 1

Re-supply Times VS Stock Distribution

Sdsu	Sgsu	Sdepot	DSU	GSU	DEPOT
0	0	0	T12341	T2342	T343
*	0	0	T12341	T2342	T343
0	*	0	T121	T2342	T343
*	*	0	T121	T2342	T343
0	0	*	T1231	T232	T343
*	0	*	T1231	T232	T343
0	*	*	T121	T232	T343
*	*	*	T121	T232	T343

Here, T_i, i + 1, ...i + j, i is the time it takes a requisition to pass from echelon i to echelon i + j while going through all echelons between, plus the time it takes the item to be shipped from echelon i + j back to i.

The echelons are numbered as follows:

1 represents DS level
2 represents GS level
3 represents depot level
4 represents rear supply

Thus, T232 is the time it takes a GSU to obtain an item from the depot given the depot has stock.

1.2.4 System Constraints

To complete the characterization of the retail system, the constraints on the system affected by stockage must be identified. Such constraints might limit the weight, volume or investment in the items in the catalogue. Because DS and GS units are required to be mobile, the constraints play an important role in the model.

In summary, the retail logistics system may be pictured as an arborescent network. The construction of the network requires knowledge of the number and type of different units in the system and the requisitioning procedures between echelons. Furthermore, the re-supply times must be defined and the various stockage constraints identified. This information is required as input for the model to be described next.

1.3 Multi-Echelon Stockage Model

1.3.1 Assumptions and Simplications

By virtue of its complexity and diversity the Army retail logistics system is difficult to model. Computer simulation models have been constructed, but their use is limited to evaluation of specific policies. The development of optimum policies through such simulators is unprofitable not only because of time and money constraints, but also because of their inability to develop general policies. If the analyst is willing to limit the scope of the study, however, it is possible to develop analytic models with which it is much easier to work. The multi-echelon stockage model is limited to looking at those factors most strongly connected with the stockage of repair parts. By limiting the factors included in the model and by making the proper assumptions, a tractable model has been built.

The following factors were felt to be important and were included in the model:

1. Probabilistic demand - Poisson

2. Re-supply times for each echelon as a function of the system stockage levels.

3. Military essentiality of the part.

4. Lowest authorized echelon of maintenance.

5. Percent of total failures of the part detected at each echelon.

6. Cost of holding an item in stock.

7. Cost of being backordered.

8. Stockage constraints

9. Transportation costs.

It is assumed that the military essentiality of each part in the catalog is known. At the present time, the development of military essentiality measures is still in its beginning stages. Thus, the military essentiality of parts is not readily obtainable today. Nevertheless, one of the primary objectives of this work is to demonstrate the possible use of military essentiality in stockage decisions. This project was conducted along with another project which developed a methodology for determining the military essentiality of repair parts.[1] The catalog on which the model was tested therefore had military essentiality measures available.

1.3.2 Cost Equation and Notation

For the present, assume it is possible to identify the cost of one unit backordered for one year. Then, the total expected cost per year of the system is expressed as

$$(1)\quad TEC(S_1,S_2,S_3) = C_3(S_1 \cdot N_1 + S_2 \cdot N_2 + S_3 \cdot N_3) + C_B[E \cdot N_1 \cdot B_{DSU}(S_1,S_2,S_3,\lambda_1) + N_2 P_2 B_{GSU}(S_2,S_3,\lambda_2)] + \lambda_3 T_r C$$

where

C_I = cost of holding one item in inventory for one year

C_B = cost of one unit being backordered for one year

E = parameter measuring the military essentiality of the part

N_1 = number of DSU's in the system

N_2 = number of GSU's in the system

N_3 = number of depots in the system

P_2 = % of total demands occurring at GSU which originate at GSU

S_1 = amount of the part stocked at an individual DSU

S_2 = amount of the part stocked at each GSU

S_3 = amount of the part stocked at each depot

λ_1 = demand rate per year of the part at a DSU

λ_2 = demand rate per year of the part at a GSU

λ_3 = demand rate per year of the part at the depot

$B_{DSU}(S_1,S_2,S_3,\lambda_1)$ = expected backorders at a DSU as a function of stock levels

$B_{GSU}(S_2,S_3,\lambda_2)$ = expected backorders at a GSU as a function of stock levels

$T_r C$ = cost of transporting one item from rear supply to the depot

Equation (1) expresses the total expected cost to the system of stocking S_1,S_2,S_3 of the item at the DSU's, GSU's, and depots respectively. The total cost is composed of three basic costs: inventory costs, backorder costs, and transportation cost. Of the three, inventory cost and transportation cost are most easily identified with "out of pocket" costs. Even though they may be difficult to determine accurately, one can be sure that money is being spent in inventory and for transportation. But such is not the case with backorder costs. It is difficult to identify a cost with backorders whether it be artificial or "out-of-pocket". In an industrial situation, a backorder can often be associated with lost profit or revenue. But the Army stocks parts to help insure the success of combat missions and a dollar value, as all analysts will agree, is difficult to assign to mission success. An attempt was made in this study, however, to establish a cost of backorders. Say an end item is returned to DSU with a failed part, and assume this failure deadlines the end item, the DSU will then exchange a good item from his float for the down item, repair the down item and finally place it in the float to be used later. When the failed part is not available for the repair action, the end item must sit waiting for the part (assuming no cannibalization). A down end item just waiting to be repaired is much like a part sitting in inventory waiting to be used. The cost of a backorder was therefore set to an inventory holding cost factor times the cost of the end item, i.e., a part backorder corresponds with a down end item waiting for repair, so assign the backorder a cost of holding an end item in "inventory". Notice that the essentiality E is not applied to B_{GSU}. This is because a failure detected at the GSU always results in the end item taken out of service.

The average backorder terms in (1) express the service the stockage is providing. Average backorders as used here measures the expected number of backorders per year. Therefore, a backorder which lasts one year receives the same value as two backorders that last 1/2 year, and so on. It is logical that the backorder measure reflect both the frequency and length of backorders. Certainly two backorders should receive a penalty greater than one backorder, but also two backorders that last a year should receive a greater penalty than two backorders which last 6 months.

The total cost equation then expresses the trade off between the out of pocket inventory and transportation costs, and the average

backorder performance cost. It is desired to find those stock levels which minimize TEC. In mathematical terms the problem is

$$\underset{S_1,S_2,S_3}{\text{Minimize}}\ TEC(S_1,S_2,S_3)$$

1.4 Computation of Average Backorders

1.4.1 Palms Theorem

To minimize $TEC(S_1,S_2,S_3)$ it must be possible to evaluate B_{GSU} and B_{DSU} for any feasible values of S_1, S_2 and S_3. However, to calculate these values several critical assumptions must be made. First it is assumed that all failure detections occur exponentially. In other words, the number of failures detected at the organizational level, the direct support units, or the general support units in a time T is Poisson distributed. Secondly, all levels follow an S,S-1 policy. Under such a policy, a unit orders a replenishment each time it receives a demand. The number of items in stock and on order is then always equal to S. Notice that these two assumptions insure that total demand at any unit in the system is Poisson. The S,S-1 policy merely passes on the same distribution it sees, and since demand originates as Poisson, only Poisson demand is experienced by each unit.

There is a queuing theorem due to Palm[2] which says that if customers arrive in a Poisson stream at a rate λ and if the service system consists of an infinite number of parallel servers each with a mean service time, T, the number of customers in the system in the steady state is Poisson with mean λT. This result is true regardless of the service time distribution.

Now consider a stockage point in the system. Conceptually, his situation is the same as the queuing situation described above. A demand corresponds to a customer arrival, and the resupply time of the replenishment order corresponds to the service time. Strictly speaking, Palm's theorem requires that the service time of each customer be independent of the other customers' times. Thus, if the queuing analogy is to hold exactly we must be willing to accept the assumption of independent resupply times.

There are two basic objections to the assumption of independent resupply times. For one, it allows the possibility of cross-over of orders, i.e., a requisition placed later than another may actually be satisfied first. For another, as will be seen later, our conception of random resupply times is based on the random stock-out periods induced by random demand. When the supplier is in stock he responds in a fixed time. When he is out of stock there is an additional random waiting time for his replenishment. The resupply times he furnishes are anything but independent. Both these problems are minimized if the item is slow moving. If it is unlikely that 2 or more demands occur during a stockout period the second problem is overcome and if the time between demands is large enough, cross overs will not occur.

1.4.2 Computational Scheme to Find Average Backorders

In computing average backorders, a top to bottom method is required. First the average backorder level at the depot is computed using Palm's theorem and then this is used to find the GSU's average resupply time. Once this is found the average backorder level is computed for the GSU's. In similar fashion the average backorders at the DSU's are found.

The average backorders of a unit can be found from the replenishment order distribution. By the S,S-1 policy we know the inventory level is $X = S - Y$ where Y is the number of orders in resupply; and thus the probability that X equals K is the same as the probability that Y equals $S-K$. But this is just the Poisson probability. An inventory system is said to be backordered K units if its inventory level (on hand-on order) is equal to $-K$. The expected backorders

$$B = \sum_{i=-1}^{-\infty} -ip(i)$$

where $p(i)$ is the probability that inventory is i. In this case

$$B = \sum_{j=s}^{\infty} (j-s)p(j,\lambda T)$$

where $p(j,\lambda T)$ is the Poisson probability that on order is j

As an example of the calculation of B_{DSU} and B_{GSU} consider the following case. The lowest echelon of maintenance for a given item is the organization. Stockage is, therefore, considered for all points in the system. While not necessary, the assumption of system symmetry is made to simplify the calculations. All units in any particular level are assumed to have identical failure rates and assumed to follow the same order policy, otherwise separate backorder calculations would be required for each stockage point. The depot stocks an amount S_3 and follows an S_3,S_3-1 policy. Furthermore, the rear supply point to which the depot goes for replenishment accomplishes resupply in an average time T_{343}. By Palm's theorem the probability of j orders in resupply is

$$P_{DEP}(j,\lambda_3 T_{343}) = \exp(-\lambda_3 T_{343})\cdot(\lambda_3 T_{343})^j/j!$$

where λ_3 is the demand rate at the depot

Now the GSU's requisition spares from the depot and they experience a resupply time which is composed of a travel and processing time plus a wait for the item at the depot. The travel and processing time is

assumed constant and is the T_{232} defined earlier. The waiting time, W_2, is a random variable which can range anywhere from zero to the maximum time it takes the depot to get replenished. Clearly W_2 depends on the amount stocked by the depot. The more the depot stocks the less the chances a customer will wait, and the average waiting time will decrease. Through use of the standard queueing equation $L = \lambda W$ it is found that the average wait, W_2, is

$$W_2 = B_{DEP}/\lambda_3$$

with B_{DEP} given by

$$B_{DEP} = \sum_{j=s_3}^{\infty} (j-s_3)p_{DEP}(j,\lambda_3 T_{343})$$

A GSU ordering from the depot then sees a total average resupply time of

$$T_2 = T_{232} + B_{DEP}/\lambda_3$$

And if it is possible to compute the GSU resupply time, it is possible to compute the probability function of the number of GSU orders in resupply.

A DSU ordering from a GSU can be analyzed in the same manner. The average replenishment time of a DSU is

$$T_1 = T_{121} + B_{GSU}/\lambda_2$$

and its average backorders are

$$B_{DSU} = \sum_{j=s_1}^{\infty} (j-s_1)p_{DSU}(j,\lambda_1 T_1)$$

The above analysis is conditional upon positive GSU and depot stockage. As a further example of the computations, suppose that the depot stocks but the GSU's do not, then the analysis between GSU and depot does not change; however, when a DSU orders from a GSU his requisition is passed on to the depot and the additional wait for parts occurs at the depot rather than the GSU. Its average resupply time is, in this case

$$T_1 = T_{1231} + B_{DEP}/\lambda_3$$

In the general situation the distribution of orders in resupply is found for a unit by identifying that level's processing and shipping time plus its wait for parts. Its average wait for parts is found from the B at the unit which ultimately supplies the replenishment items. There is no wait included in the resupply time when the rear supply is the ultimate supplier, for the average wait at the rear is already included in the estimate of T_{343}. Tables 2 and 3 present average resupply times for all stock policies. When the average resupply time has been determined, Palm's theorem is employed to find the steady state inventory distribution. By the nature of the relationships between echelons, the analysis must start at the depot and proceed downward to the DSU's. Using the procedure it is possible to compute TEC for any stockage levels S_1,S_2, and S_3.

1.4.3 Transportation Cost

The remaining term in the total cost equation to be evaluated is the transportation cost. Only the rear supply to depot transportation cost is considered. Not only is this the largest portion of the total transportation cost but also is the only portion of the transportation cycle where a distinct comparison between modes can be made.

For much of the intra-theater shipments there is no real choice of a transportation mode, but there is always the choice of shipping by air or by sea from the rear supply to the theater depot. Both of these modes have their own distinct characteristics and for routine replenishment actions it is not obvious which mode should be used. The transportation mode should be selected on the basis of the mode's cost and its effect on the system's inventory and backorder cost. The air mode, for example, will require less inventory than the sea mode but will cost more to use. It will only be worthwhile if it reduces inventory sufficiently to compensate for the increased transportation cost. Thus, for each mode the optimum stock levels are computed and the inventory and backorder costs for these levels added to the transportation cost. On the basis of this total cost the optimum transportation mode and stockage level are selected.

We have shown how it is possible to evaluate the total cost of stocking any amounts at the DSU's and GSU's and depots and have indicated how the best stock levels and transportation modes are selected. The next chapter presents the algorithm which finds those stock values which minimize the total cost.

Table 2

Expected Resupply Time

Lowest Authorized Echelon of Maintenance is Organization or DSU

S_1	S_2	S_3	T_1	T_2	T_3
0	0	0	T12341	T2342	T343
≠	0	0	T12341	T2342	T343
0	≠	0	$T121+B_{GSU}/\lambda_2$	T2342	T343
≠	≠	0	$T121+B_{GSU}/\lambda_2$	T2342	T343
0	0	≠	$T1231+B_{DEP}/\lambda_3$	$T232+B_{DEP}/\lambda_3$	T343
≠	0	≠	$T1231+B_{DEP}/\lambda_3$	$T232+B_{DEP}/\lambda_3$	T343
0	≠	≠	$T121+B_{GSU}/\lambda_2$	$T232+B_{DEP}/\lambda_3$	T343
≠	≠	≠	$T121+B_{GSU}/\lambda_2$	$T232+B_{DEP}/\lambda_3$	T343

Table 3

Expected Resupply Time

Lowest Authorized Echelon of Maintenance is GSU

S_1	S_2	S_3	T_1	T_2	T_3
•	0	0	•	T2342	T343
•	≠	0	•	T2342	T343
•	0	≠	•	$T232+B_{DEP}/\lambda_3$	T343
•	≠	≠	•	$T232+B_{DEP}/\lambda_3$	T343

CHAPTER II

OPTIMIZATION ALGORITHM

The algorithm to be described presently was used on a catalog of 710 items and produced the stockage levels in a time of approximately one and one half minutes when programmed in Fortran IV on a GE 635 computer. This amounts to 0.126 seconds on the average per item. Empirical studies on the algorithm indicate that it will reach an exact optimum for a large class of items. However, for items characterized by large failure rates the algorithm will at worst achieve values in the neighborhood of the true optimum. Fortunately, these are the items which are insensitive to small changes from the true optimum and coming close, therefore, results in a negligible loss in cost. Furthermore, high demand items are rare among typical repair parts. Though this algorithm was developed from heuristic arguments which are not always valid, its performance, nevertheless, justifies its acceptance.

Restating the problem mathematically, assuming N_3 equal to 1, it is desired to minimize

$$\begin{aligned} TEC(S_1,S_2,S_3) &= C_I(S_1 N_1 + S_2 N_2 + S_3) \\ &\quad + C_B[E N_1 B_{DSU} + N_2 B_{GSU}] \end{aligned}$$

over the three stock variables S_1, S_2, and S_3 where B_{DSU} and B_{GSU} are of the form

$$B = \sum_{j=s}^{\infty} (j-s)p(i,\lambda T)$$

The transportation cost has been omitted from this equation because it cannot be controlled by the decision variables S_1,S_2, and S_3.

The standard approach to this optimization problem would be to establish the inequalities which must be satisfied at the optimum by the difference equation approach. It is not clear, however, that this approach would always find the true minimum and not just a local minimum. To insure a minimum, eight inequalities would need to be satisfied, and moreover, no obvious plan of attack is suggested by this procedure. As it happens, it is easier to develop an optimization algorithm by taking advance of the "top to bottom" resupply interactions between echelons.

Notice that stockage quantities at the upper levels affect the stockage decision at the lower levels only through the expected resupply times. This means that if the depot and GSU stockage quantities are fixed, the DSU's average resupply time remains fixed also. Let $TEC(S_1 S_2^*,S^*)$ be the

total expected cost as a function of S_1 when S_2 and S_3 are fixed at S_2* and S_3* respectively. By virtue of the fact that $TEC(S_1S_2*,S_3*)$ is convex, minimization of $TEC(S_1S_2,S_3*)$ over S_1 is a simple matter. Convexity or, in other words, diminishing marginal returns of stockage is an expected, but not necessary property of inventory. In this one dimensional case, it indeed can be shown to hold. But, what about GS and depot stockage? Is the property of diminishing marginal returns valid for this stockage too? First consider GS stockage. Clearly, the returns from GS stockage depend also on the depot and DS stockage decisions. If, as before, the depot stockage is fixed at S_3*, then only the DS stockage policy affects the returns from GS stock. When GS stockage is changed, the DSU average resupply time is altered. An alert DSU will readjust its stock position to its new resupply time. Its reaction, of course, partly determines the returns to GS stockage decisions. Thus, to determine the returns to GS stockage, a DS policy must be defined. But since we are concerned with minimizing costs, the most logical DS policy is a policy which minimizes his cost. This suggests the following question:

If $TEC(S_1/S_2/S_3*)$ is the minimum cost of $TEC(S_1S_2,S_3*)$ is $TEC(S_1/S_2/S_3*)$ convex with respect to S_2?

To answer this question we must determine the effect of S_2 on B_{GSU} and B_{DSU} given that S_3 equals S_3*. Clearly S_2 produces diminishing marginal returns to B_{GSU}, but the effect on $B_{DSU}(S_1)$ is not so obvious. As S_2 is changed, the stockage policy at the DSU's is reevaluated to find a new S_1. S_1 is found from the inequalities

$$E\,P(S_1+1,U_1) \leq \frac{C_I}{C_B} \leq E\,P(S_1U_1)$$

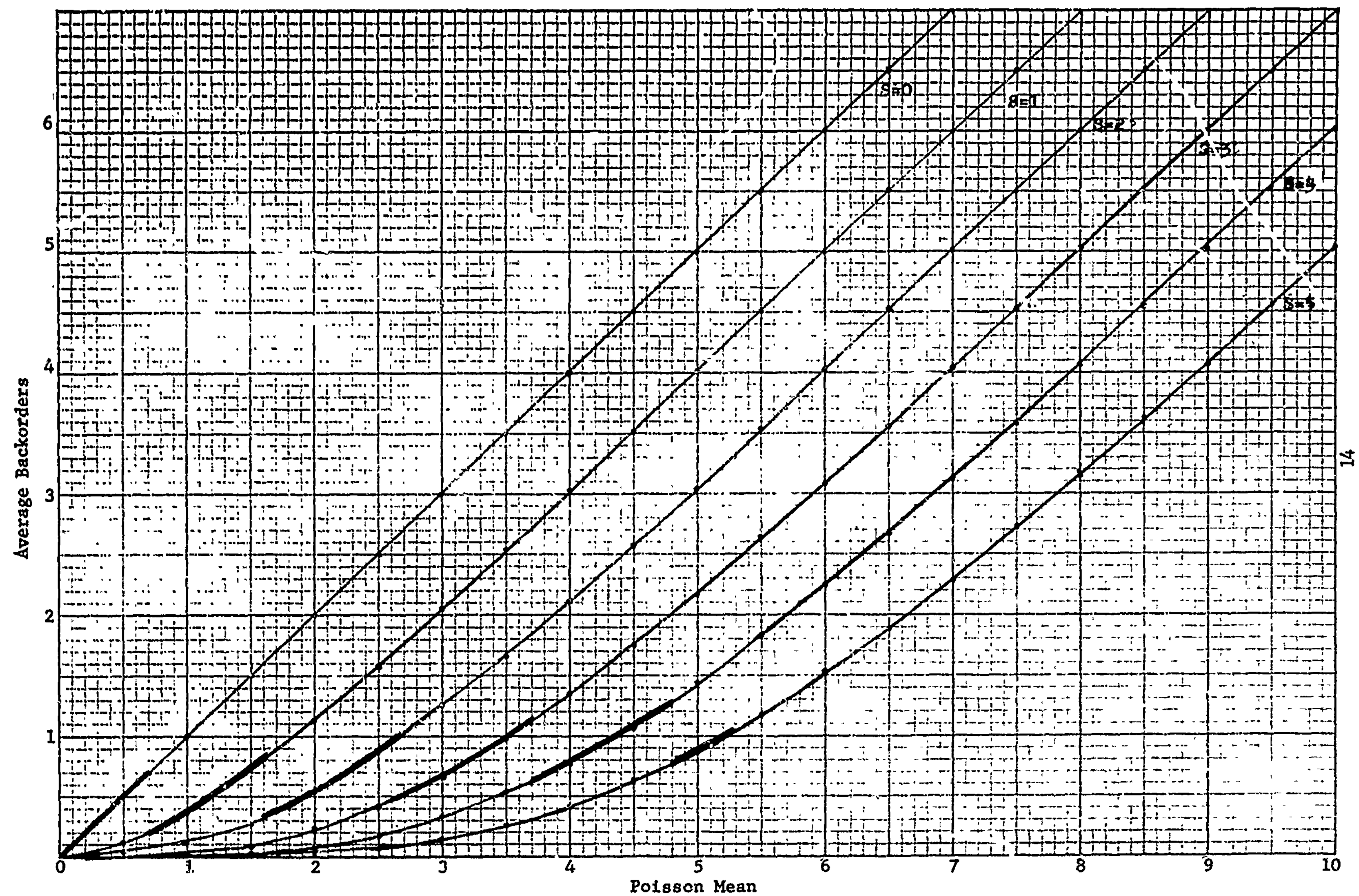
where

$$P(S_1U_1) = \sum_{j=s}^{\infty} \exp(-u_1)\, u_1{}^j/j!$$

and $U_1 = \lambda_1(T_1+B_{CSU}/\lambda_2)$ is the expected mean. $P(S_1,U_1)$ is merely the difference in B_{DSU} when stocking S or S-1.

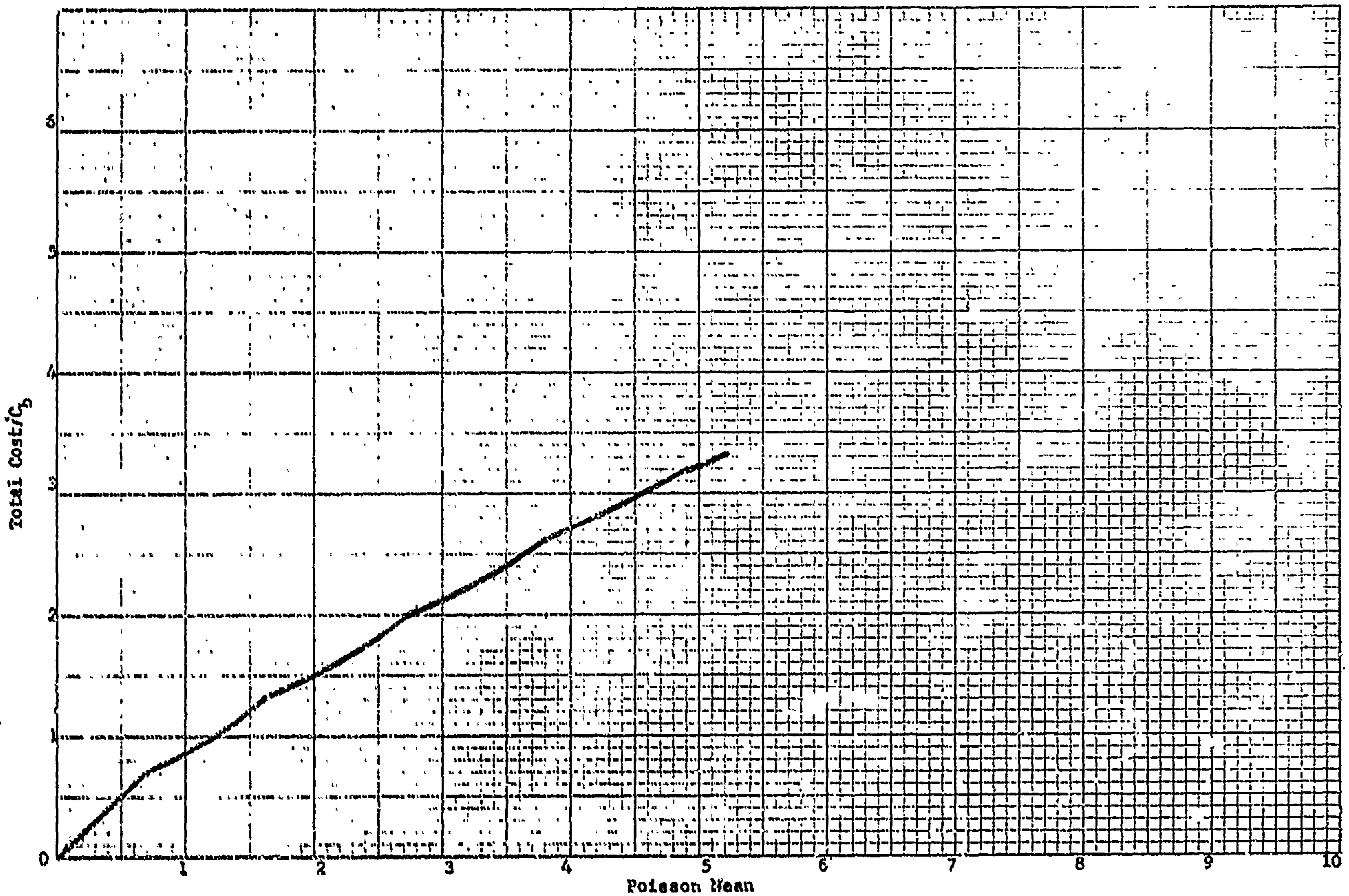
Figure 2 presents a plot of B_{DSU} versus U_1 for several values of S. For the purpose of discussion, and with no loss in generality assume for a value U_1 the optimum stock S_1 is 5. As U_1 decreases, the optimum level will remain at 5 until the difference in B_{DSU} of stocking 4 to stocking 5 is equal to $C_I/E\cdot C_B$ whereupon the optimum stock quantity will become 4. In the same manner, the optimum level will remain at 4 until the difference in B of stocking 3 or 4 is again $C_I/E\ C_B$ when it will become 3. A curve can therefore be traced out as in Figure 2 which defines the optimum stock values S_1, for all values of U_1. Furthermore, from this curve it is possible to construct the curve of Figure 3 which plots the DSU expected cost vs U.

FIGURE 2



S=0
S=1
S=2
S=3
S=4
S=5
Average Backorders
Poisson Mean
0
1
2
3
4
5
6
7
8
9
10

FIGURE 3



$$TC = C_I N_1 S_1 + E\, C_B N_1 B_{DSU}(S_1)$$

At the breakpoints of the curve in Figure 2, the additional inventory cost just offsets the reduction in backorder cost. For example at the value U_1 the optimum stock level changes from 3 to 4. Inventory costs rise by C_I and backorders costs are reduced by C_I. Thus, to construct the expected cost curve eliminate the discontinuities by tacking the section of the curve for $S_1 + 1$ onto the end of the curve for $S_1{}^*$ and multiply the resulting curve by $C_B\, E\, N_1$ to get the proper units.

Now as S_2 changes, what we have called U_1 changes by $\frac{\lambda_1}{\lambda_2} \Delta B_{GSU}$ where B_{GSU} signifies the change in B_{GSU} with change in S_2. Thus, the property of diminishing marginal returns holds with respect to U_1. Unfortunately, it cannot be concluded with certainty that the DSU optimum costs displays this property. Note the scalloped appearance of the curve of Figure 3. This was not drawn by accident, but will always occur. Where the sections are connected, the left hand section will have a slope greater than the right hand section. Because of this scalloped nature, it is not inconceivable that DSU optimum cost will not display the property of diminishing marginal returns. Nevertheless, from purely heuristic considerations, it is possible to argue that this property will be observed. U_1 as was noted, is strictly convex with respect to S_2. It is, therefore, possible that convexity will be imposed on DSU optimum cost by the convexity of U_1. It is not only possible, but, in fact, has been verified empirically for a large class of items.

Working on the assumption then that $TEC(S_1/S_2/S_3{}^*)$ is convex over S_2, it is easy to find

$$\min_{S_2} TEC(S_1/S_2/S_3{}^*)$$

Similarly, the same convex property is desired of S_3. As before, a GS policy and a DS policy must be defined. First the GSU reacts to the change in S_3 and then the DSU reacts to the GSU's change in an optimal way. The GSU's optimal policy includes his effect on the DSU's. Thus we ask: "is

$$TEC(S_1, S_2, S_3) = \min_{S_2} TEC(S_1/S_2/S_3)$$

convex with respect to S_3?"

Again as before we are led to conclude that it is in most cases but is not a certainty.

From these considerations, an optimization algorithm was developed. In essence the algorithm does the following. Depot stock, S_3, is initialized at zero. The values of S_2 and S_1 which give the minimum cost for fixed S_3

are found. S_3 is incremented until the optimum cost does not improve. The values of S_3, S_2, S_1 which give the minimum cost at that time are declared the optimum stock levels.

Because the mathematical foundation of the algorithm is not entirely firm, the algorithm was tested prior to its use to see how well it performed. A comparison of the results was made with those from another algorithm which always found the true optimum though often with much time. The results of this test indicated the algorithm would achieve the true optimum for all but high demand items. It is not difficult to understand why the algorithm does not achieve the true optimum for those items. We argued earlier that convexity on S_2 and S_3 depended upon B_{DEP}/λ_3 and B_{CSU}/λ_2 imposing convexity by virtue of the strength of their own convexity. But for the high demand items average backorders does not strongly display the property of diminishing marginal returns. Thus, a convex behavior would not necessarily be expected. Fortunately, the same factors which prevent a true optimum from being achieved are also the factors which redeem the algorithm. The high demand items are insensitive to small changes from the optimum. To be successful, therefore, the algorithm need only produce stock levels in the neighborhood of the optimum. And in the test runs the algorithm always came within one of the true optimum levels.

Overall then the algorithm must be judged satisfactory. It is very fast, but despite its speed is able to produce optimum or nearly optimum stockage levels.

CHAPTER III

3.1.1 The Multi-Item Problem

Until now the discussion has only considered the development of a single item model; but, when considering the retail system the more realistic approach is to develop stockage policies for many items accounting for the interactions among the items. Because there are constraints on stockage, the item characteristics (price, failure rate, weight, etc.) should be used to determine the best selection of item stockage levels from the entire catalog. Under the constrained case, the stockage values which provide the best performance cannot be selected without consideration of the item characteristics involved in the constraints.

The constrained problem is formulated as follows: Suppose there are N items to be considered for stockage in the system. $TEC_i(S_{1i},S_{3i})$ is the cost of stocking S_{1i}, S_{2i}, and S_{3i} of item i at the DSU's, GSU's and depot respectively, then the cost for the entire catalog is

$$TEC(S) = \sum_{i=1}^{N} TEC_i(S_{1i},S_{2i},S_{3i})$$

Further suppose that there are M constraints on stockage of the form

$$G_j(S) \le b_j \qquad j = 1,2,\ldots M$$

The optimum stockage levels are found be solving the mathematical problem

$$\text{MIN } TEC(S)$$

$$\text{subject to} \qquad G_j(S) \le b_j$$

This multi-item formulation does not consider many of the interactions between items. In particular, no attention is given to joint ordering policies; each item is still managed independently. Rather, only an attempt is made to insure that the stocking point will not violate any of several constraints on its total repair part stockage. This is important because it means that the form of the policy is still the same as that of the single item case. Each point stocks some amount and orders whenever it gets a demand. Furthermore, because each item is managed independently, the possibility of decomposing the N item problem into N single item problems similar to the one discussed in the previous two chapters might be feasible. In fact, unless this can be done, optimization is virtually impossible since

the number of variables to be found is likely to be greater than 3500. Fortunately, for those constraint functions $G_j(S)$ of interest, decomposition is indeed possible. We are interested in weight, volume, and investment constraints at any or all of the echelons. For these constraints $G_j(S)$ is of the form

$$G_j(S) = \sum_{i=1}^{N} \sum_{k=1}^{3} \alpha_{jik} S_{ik}$$

where

i identifies the item

k identifies the echelon

j identifies the constraint parameter for item i

For example α_{ji} might be the weight, or price, of item i, in which case $G_j(S)$ would be the total weight or investment of the stockage catalog. If the constraint is to be over one echelon only, the summation over all k would be omitted.

3.1.2 Lagrange Multiplier Formulation

The method used to solve the constrained problem was the method of generalized Lagrange multiplier (GLM). With this method a Lagrangian function is formed as

$$L(\underline{S},\lambda) = TEC(\underline{S}) + \sum_{j=1}^{M} \lambda_j [G_j(\underline{S}) - b_j]$$

and the following problem is solved

$$\begin{array}{l} \text{MIN } L(S,\lambda_o) \\ \underline{S} > o \end{array}$$

where λ_o represents a known vector of Lagrange multipliers. Let the solution for a particular set of λ's be $S^*(\lambda_o)$ Then $S^*(\lambda_o)$ simultaneously solves the problem

$$\text{MIN } TEC(S)$$

subject to $G_j(S) \le G_j(S^*(\lambda_o))$

Thus, if a λ_o can be found such that $G_j(S^*(\lambda_o)) = b_j$ the original constrained problem has been solved. Moreover, since

$$TEC(\underline{S}) = \sum_{i=1}^{N} TEC_i(\underline{S}_i)$$

and

$$G_j(S) = \sum_{i=1}^{N}\sum_{k=1}^{3} \alpha_{ji}S_{ik}$$

then

$$L(\underline{S},\underline{\lambda}) = \sum_{i=1}^{N} \left\{TEC(S_{1i},S_{2i},S_{3i}) + \sum_{j=1}^{M}\lambda_j \sum_{k=1}^{3} \alpha_{ji}S_{ik}\right\} - \sum_{j=1}^{M} \lambda_j b_j$$

and the minimization can be accomplished by

$$\underset{S_{1i},S_{2i},S_{3i}}{\mathrm{MIN}} \quad TEC(S_{1i},S_{2i},S_{3i}) + \sum_{j=1}^{N} \lambda_j \sum_{k=1}^{3} \alpha_{ji}S_{ik}$$

for each i. In that case, only three variables need be found for each minimization.

3.2.1 A Proposed Solution Technique

Notice that for any particular set of λ's,

$$\sum_{j=1}^{N}\lambda_3 \sum_{k=1}^{3} \alpha_{ji}S_{ik}$$

appears as an additional holding cost. Effectively then, the above is identical with the single item problem whose solution was presented in Chapter 2. In other words, the algorithm already developed for the single item problem can also be used for the multi-item problem for any particular set of multipliers. The primary burden of the multi-item solution scheme then is to determine the multipliers λ_o which yield $G_j(S^*(\lambda_o)) = b_j$.

If the functional relationship were known between S* and the $\underline{\lambda}$, there would be little difficulty in determining $\underline{\lambda}_o$. But because this relationship is not known, it is no simple matter to find $\underline{\lambda}_o$ even when the number of constraints is small.

Consider the basic problem. A set of multipliers is desired which produce a specified result. The mathematical relationship between the λ and their resulting outcome cannot be expressed literally. Nevertheless, the outcome can be measured for any particular values of λ via computer solution. This problem is in the class of problems which are normally solved with search techniques.

In essence, search techniques are primarily sequential techniques which select a new point to search on the basis of the results from the previous points searched. But the difficulties of such a scheme when applied to this problem are many. In the first place, to measure the relationship between a particular set of multipliers and the amounts of resources used requires a complete optimization over the entire catalog. This is not bad however, unless the number of optimizations required to find the λ_o is great. When there is only one constraint and, of course, only one multiplier, it is not difficult to converge to λ_o. With more than one constraint convergence becomes slower and slower. Rand has developed a linear programming procedure to find λ_o with which they have had considerable success. They have applied the procedure to similar problems achieving a solution in an acceptable amount of time. There is little doubt that Rand's procedure would be applicable to this problem too. Nevertheless, we wished to try a scheme of our own, conceived after a visit with Dr. George Pugh of the Lambda Corporation.*

Dr. Pugh devised a procedure based on statistical estimation from calculations on random samples from the population. When the elements of the population are large in number and have varying characteristics such that a few elements do not dominate the solution then Dr. Pugh's scheme can be used. Since our item catalog met these conditions, we decided to devise a procedure using the idea of random sampling.

At the present time, the procedure has not been sufficiently debugged to permit complete evaluation. It is felt, however, that the basics of the procedure are sound and that they are worthwhile reporting.

3.2.2 Multi-Item Constraint Algorithm

To begin, the items in the catalog are arranged randomly on a circular file. A guess is made of the λ value to determine a starting point (λ=o is a good starting point since it points out immediately the probable non-binding constraints). With this value of λ the optimization algorithm is run on the first R items, and the amount of resources consumed by these R items is recorded. (R is selected by the analyst). This of course provides an estimate of the resource usage rate for this λ. If these usage rates appear "correct", i.e., the rate is within some given percent at the desired rate, the λ is used for computations on the entire catalog. When this

*We are indebted to Dr. Pugh for his ideas on this approach and extend our appreciation to him.

computation does not produce satisfactory resource consumption, the algorithm is begun again using the current λ as the base point on a different sample set.

Assume the first λ chosen does not produce correct resource consumption estimates. Thus, we need to choose another multiplier using the information gained to move toward the correct λ_o. The mechanism employed to produce movement is a gradient search technique used with a pseudo-function which relates the degree to which the current estimates of resource expenditure meet the desired rates.

The precise form of the pseudo-function is not critical and should really be determined with regard to the problem being solved. There are, however, some properties the pseudo-function should have. Along a line in the λ space, thepseudo-function should be unimodal to allow a search along the line. Secondly, a unique value of the pseudo-function should correspond to the precise satisfaction of all the constraints. Finally, the pseudo-function should be able to detect even the smallest changes in the resource usage rate estimate. The pseudo-function we used was

$$F = \sum_{j=1}^{M} W_j \left| b_j^E - b_j \right|$$

where b_j^E is the current estimate of the resource consumption and W_j is a weighting factor reflecting the relative important of satisfying constraint j.

Given a base point value of λ and a sample of items, the gradient of the pseudo-function is estimated by individually perturbing the λ and measuring the change in thepseudo-function. If $\Delta^j F/\Delta\lambda_j$ is the ratio of the change in the pseudo-function due to a change in λ_j then the gradient is estimated as

$$\nabla F^E = \left(\frac{\Delta^1 F}{\Delta\lambda_1}, \frac{\Delta^2 F}{\Delta\lambda_2}, \ldots, \frac{\Delta^M F}{\Delta\lambda_M} \right)$$

The objective we had was to minimize the pseudo-function. Thus, a search is made along the negative gradient for the minimum value of the pseudo-function. When this point is found it becomes the new base point, a new sample of items is selected and the procedure continues. When the pseudo-function is acceptable the current value of λ is tested on the entire catalog of items. Note that for every base point, a new sample of items is selected from the circular file. They are picked up in sequence as they occur on the file. Every time the sample changes, so do the statistical properties of the sample. Whatever a particular λ meant to the previous sample in terms of resource consumption, it surely will not mean the same thing to the new sample because of the difference in item characteristics; but certainly the old estimates carry information and should not be thrown away.

Consider the method of estimating by exponential smoothing. This procedure is used to estimate the parameter in a process of the form.

$$Y_t = A_t + E_t$$

where E_t is a random factor.

The parameter A_t changes with t. In this problem A_t is a resource consumption rate and E_t is the random element due to sample characteristics. The index "t" identifies the particular sample and the multiplier values used on it. Exponential smoothing is a heuristic technique for estimating A_t using the equation

$$A_t = \alpha\lambda_t + (1-\alpha)A_{t-1}$$

for

$$0 \leq \alpha \leq 1$$

"α" is called the smoothing constant and is the factor which determines the relative importance assigned to the new and old observations in the current estimate. Exponential smoothing is an appealing technique for it allows information from past samples to play a part in the current decision. Some of the variability of a single sample has been eliminated with no penalty in computation time.

The estimates b_j^E which are used by the pseudo-function are exponential smoothing estimates. Depending on the choice of smoothing constant the sensitivity of the pseudo-function to the results of a single sample can be varied. Convergence to the correct multipliers thus depends strongly on the smoothing constant.

Currently, it is not possible to report complete satisfaction with the constraint technique. Some study has been done in an attempt to determine the optimum sample size and the most suitable smoothing constant. While work on the subject is still in progress, the work already done has been encouraging. The basic methodology appears to be sound. Most of the faults seem to occur because of over-programming in an attempt to insure convergence. Much of the sensitivity has been destroyed by these attempts and, as a result, movement of the multipliers is too slow. The difficulties encountered however do not seem insurmountable and we hope to continue our research in the area.

CHAPTER IV

RESULTS

4.1 General

In conjunction with another study done by this office entitled "Military Essentiality Coding", the multi-item model was tested on a catalog of 710 parts of a Baker model Rough Terrain Forklift. The essentiality project developed the military essentiality ranking for the repair parts of the forklift. Of all these parts, there were 710 on which the demand and physical characteristics (weight, volume, price) could be obtained. With these items composing the catalog, the multi-echelon model was applied to 4 retail logistics systems. Two of the systems were constructed to correspond as closely as possible to the European theater. They differed only by the transportation mode employed; one used the sea mode, the other air. Similarly there were sea and air models of the Vietnam theater.

The purpose of the test was -

1. Evaluate the effect of transportation mode on optimum inventory costs and thereby determine the optimum transportation mode.

2. Demonstrate how military essentiality measures can be used in stockage decisions.

3. It must be noted that the overriding goal of this study was not directed toward implementation of any new procedures. Rather, it was a continuation of research in multi-echelon modeling begun by the IRO. The primary goal of developing feasible computation schemes for the multi-echelon model has already been discussed in the previous chapters. Were it not for the military essentiality project the study would have ended here. An interest developed in the ways the military essentiality rankings could be used for stockage decisions. Therefore, an attempt was made to use these rankings with the multi-echelon model. Moreover, at the same time, an interest also developed for ways to select the best transport mode for an item. As a result a transportation cost algorithm was obtained from Research Analysis Corporation and the multi-echelon model was used on two transport modes, sea and air.

The results presented shortly are those which proved to be the most interesting derived from the test.

4.2 Catalog Characteristics

As mentioned previously, data was obtained on 710 items of the rough terrain forklift. Before presenting results, it will be of value to give a characterization of the catalog. Table 5 presents a frequency count of the items by price and table 6 by demand. Demand is the total number of

demands for the item during a one year period obtained from TAERS and Redball data.

TABLE 5

No. Items VS Price

Price $	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	> 50
No. Items	547	48	41	33	15	7	5	1	2	2	9

TABLE 6

No. Items VS Demand

Demand	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	>100
No. Items	543	25	28	22	18	4	3	3	7	4	53

Tables 7 and 8 further characterize the items by dollar value of demand per year and military essentiality for Germany and Vietnam. Here military essentiality class 1 is the least essential and 4 the most essential. The dollar value classes are:

I Dollar Value < 25

II 25 < Dollar Value < 250

III Dollar Value > 250

TABLE 7

Catalog Characteristics

Germany

DV/MEC	1	2	3	4	Totals
I	14	16	169	405	604
II	6	0	10	69	85
III	0	0	2	19	21
TOTALS	20	16	181	493	710

TABLE 8

Vietnam

DV/MEC	1	2	3	4	Totals
I	14	15	169	393	591
II	6	1	8	75	90
III	0	0	4	25	29
TOTALS	20	16	181	493	710

4.3 Optimum Policy Characteristics

The following tables (9,10,11,12) summarize the results of the tests by dollar value of demand, military essentiality, and stockage policy. The triplet S_1,S_2,S_3 is used to represent the optimum stock at DSU, GSU, and depot respectively, and * is used to designate some positive stock. The stockage policies have been classified as follows:

Policy	Class
0,0,0	I
*,0,0	II
*,0,0	III
,,0	IV
0,0,*	V
,0,	VI
0,*,*	VII
,,*	VIII

Notice that no items prefer policies II or VI. Whenever stock is placed at the DSU level, it is always supported by GSU stockage and a good portion of the time by depot stockage. This seems to verify the policy in use today that stockage at a level be supported by stockage at the next higher level. The same verification is not evident of GSU support stockage. Around 50% of the items with GSU stockage are not backed up by depot stockage.

The results in the table show nothing unexpected. For example, in going from the sea mode to the air mode there is a definite shift away fr m

TABLE 9

Germany Sea

Policy Class

DV	MEC	I	II	III	IV	V	VI	VII	VIII	Total
I	1	0	0	5	0	3	0	6	0	14
	2	0	0	10	0	0	0	2	4	16
	3	0	0	53	28	3	0	13	72	169
	4	0	0	76	109	73	0	41	106	405
TOTAL		0	0	144	137	79	0	62	182	604
II	1	0	0	0	0	0	0	6	0	6
	2	0	0	0	0	0	0	0	0	0
	3	0	0	1	0	0	0	2	7	10
	4	0	0	0	1	0	0	10	58	69
TOTAL		0	0	1	1	0	0	18	65	85
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	0	0	1	0	0	0	3	15	19
TOTAL		0	0	1	0	0	0	3	17	21

TABLE 10

Vietnam Sea

Policy Class

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	0	0	4	0	4	0	0	0	14
	2	0	0	10	0	0	0	3	2	15
	3	0	C	66	6	4	0	17	76	169
	4	0	0	85	87	83	0	38	100	393
TOTAL		0	0	165	93	91	0	64	178	591
II	1	0	0	0	0	0	0	6	0	6
	2	0	0	0	0	0	0	1	0	1
	3	0	0	0	0	0	0	2	6	8
	4	0	0	0	0	0	0	15	60	75
TOTAL		0	0	0	0	0	0	24	66	90
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	1	0	0	3	4
	4	0	0	1	0	0	0	3	21	25
TOTAL		0	0	1	0	1	0	3	24	29

TABLE 11

Germany Air

Policy Class

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	1	0	7	0	3	0	3	0	14
	2	0	0	11	3	0	0	1	1	16
	3	0	0	64	65	5	0	0	35	109
	4	2	0	107	160	73	0	9	54	405
TOTAL		3	0	189	228	81	0	13	90	604
II	1	0	0	0	0	0	0	6	0	6
	2	0	0	0	0	0	0	0	0	0
	3	0	0	1	0	1	0	1	7	10
	4	0	0	3	14	0	0	7	45	69
TOTAL		0	0	4	14	1	0	14	52	85
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	0	0	1	0	0	0	3	15	19
TOTAL		0	0	1	0	0	0	3	17	21

TABLE 12.

Vietnam Air

Policy Class

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	1	0	6	0	3	0	4	0	14
	2	0	0	12	0	1	0	0	2	15
	3	0	0	72	46	14	0	1	36	169
	4	2	0	104	131	92	0	8	56	393
TOTAL		3	0	194	177	110	0	13	94	591
II	1	0	0	0	0	0	0	6	0	6
	2	0	0	0	0	0	0	1	0	1
	3	0	0	1	0	0	0	1	6	8
	4	0	0	3	9	0	0	12	51	75
TOTAL		0	0	4	9	0	0	20	57	90
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	1	0	0	3	4
	4	0	0	1	0	0	0	3	21	25
TOTAL		0	0	1	0	1	0	3	24	29

TABLE 13

Percent of Demands Detected at GSU = 0

Germany Sea

DV	MEC	I	II	III	IV	V	VI	VII	VIII	Total
I	1	7	0	0	0	3	0	4	0	14
	2	0	3	4	1	6	0	1	1	16
	3	0	66	17	27	19	15	12	13	169
	4	0	99	50	57	73	43	41	39	402
TOTAL		7	168	71	85	101	58	58	53	601
II	1	2	0	0	0	1	0	2	1	6
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	1	1	0	2	6	10
	4	0	0	0	23	0	1	10	36	70
TOTAL		2	0	0	24	2	1	14	43	86
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	1	0	0	2	0	0	3	15	21
TOTAL		1	0	0	2	0	0	3	17	23

TABLE 14

Percent of Demands Detected at GSU = 0

Vietnam Sea

DV	MEC	I	II	III	IV	V	VI	VII	VIII	Total
I	1	6	0	0	0	4	0	4	0	14
	2	0	2	1	1	10	1	0	0	15
	3	0	70	19	19	27	7	12	14	168
	4	0	143	47	37	91	6	37	28	389
TOTAL		6	215	67	57	132	14	53	42	586
II	1	1	0	0	1	2	0	2	0	6
	2	0	0	0	0	0	0	1	0	1
	3	0	0	0	3	0	0	2	4	9
	4	0	0	0	18	0	5	14	39	76
TOTAL		1	0	0	22	2	5	19	43	92
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	1	1	0	0	2	4
	4	1	0	0	6	0	0	3	18	28
TOTAL		1	0	0	7	1	0	3	20	32

TABLE 15

Percent of Demands Detected at GSU = 0

Germany Air

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	10	1	3	0	0	0	0	0	14
	2	2	4	5	1	4	0	0	0	16
	3	0	85	29	23	19	12	0	1	169
	4	2	168	80	51	73	10	9	9	402
TOTAL		14	258	117	75	96	22	9	10	601
II	1	3	0	0	0	2	1	0	0	6
	2	0	0	0	0	0	0	0	0	0
	3	0	0	1	1	1	1	1	5	10
	4	0	2	3	33	0	6	7	19	70
TOTAL		3	2	4	34	3	8	8	24	86
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	1	0	0	7	0	0	3	10	21
TOTAL		1	0	0	7	0	0	3	12	23

TABLE 16

Percent of Demands Detected at GSU = 0

Vietnam-Air

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	9	0	1	0	4	0	0	0	14
	2	2	3	1	0	8	1	0	0	15
	3	0	77	24	11	33	19	1	3	168
	4	2	151	73	39	92	19	8	5	389
TOTAL		13	231	99	50	137	39	9	8	586
II	1	3	0	0	0	2	0	1	0	6
	2	0	0	0	0	1	0	0	0	1
	3	0	0	0	3	1	1	1	3	9
	4	0	5	2	21	0	12	12	24	76
TOTAL		3	5	2	24	4	13	14	27	92
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	2	1	0	0	1	4
	4	0	0	0	9	0	3	3	13	28
TOTAL		0	0	0	11	1	3	3	14	32

TABLE 17

Percent of Demands Detected at GSU = 0

Germany - Optimal Mix

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	10	0	2	0	0	0	2	0	14
	2	2	4	5	0	4	0	0	1	16
	3	0	85	28	20	19	9	1	7	169
	4	2	166	75	37	73	7	14	28	402
TOTAL		14	255	110	57	96	16	17	36	601
II	1	2	0	0	0	k	0	2	1	6
	2	0	0	0	0	0	0	0	0	0
	3	0	0	1	1	1	1	1	5	10
	4	0	2	2	27	0	4	8	27	70
TOTAL		2	2	3	28	2	5	11	33	86
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	1	0	0	4	0	0	3	13	21
TOTAL		1	0	0	4	0	0	3	15	23

TABLE 17-A

Percent of Demands Detected at GSU = 0

Germany - Optimal Sea

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	2	0	0	0	0	0	2	0	4
	2	0	0	3	0	0	0	0	1	4
	3	0	9	6	8	0	0	1	6	30
	4	0	6	10	17	4	0	10	24	71
TOTAL		2	15	19	25	4	0	13	31	109
II	1	1	0	0	0	1	0	2	1	5
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	4	4
	4	0	0	0	13	0	0	6	21	40
TOTAL		1	0	0	13	1	0	8	26	49
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	2	2
	4	1	0	0	2	0	0	2	13	18
TOTAL		1	0	0	2	0	0	2	15	20

TABLE 18

Percent of Demands Detected at GSU = 0

Vietnam - Optimal Mix

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	9	0	0	0	3	0	2	0	14
	2	2	3	1	1	8	0	0	0	15
	3	0	75	29	10	27	15	2	10	168
	4	1	150	70	32	91	11	13	21	389
TOTAL		12	228	100	43	129	26	17	31	586
II	1	2	0	0	1	1	0	2	0	6
	2	0	0	0	0	0	0	1	0	1
	3	0	0	0	2	1	1	1	4	9
	4	0	5	2	14	0	8	12	35	76
TOTAL		2	5	2	17	2	9	16	39	92
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	1	1	0	0	2	4
	4	0	0	0	6	0	2	3	17	28
TOTAL		0	0	0	7	1	2	3	19	32

TABLE 18-A

Percent of Demands Detected at GSU = 0

Vietnam - Optimal Sea

DV	MEC	I	II	III	IV	V	VI	VII	VIII	TOTAL
I	1	2	0	0	0	1	0	2	0	5
	2	0	2	1	1	0	0	0	0	4
	3	0	68	19	6	3	2	1	8	107
	4	0	138	27	22	13	0	10	20	230
TOTAL		2	208	47	29	17	2	13	28	346
II	1	1	0	0	1	1	0	2	0	5
	2	0	0	0	0	0	0	1	0	1
	3	0	0	0	1	0	0	0	4	5
	4	0	0	0	10	0	0	9	29	48
TOTAL		1	0	0	12	1	0	12	33	59
III	1	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0
	3	0	0	0	1	0	0	0	2	3
	4	0	0	0	4	0	0	2	15	21
TOTAL		0	0	0	5	0	0	2	17	24

depot stockage. The data indicates that *,*,* policies go to *,*,0 policies and 0,*,* go to 0,*,*. There are some exceptions, of course, but these are few. Also a strong relationship exists between the results for the theaters with the same transport mode. Depot stockage for Vietnam is more preferable due to the greater number of units the depot supports.

The support stockage noted in the above paragraph was bothersome. We felt the reason for stock held at the GSU when also held at the DSU might result only because demands were generated at the GSU as well as the DSU. Therefore, the runs were repeated with no demands generated at the GSU. Tables 13, 14, 15, 16 indicate the results of these runs. No longer is GSU backup stock required.

Also for this run similar tables 17 and 18 are shown for the optimal mix of policies. A further breakdown is shown in tables 17-A and 18-A. These tables show the sea mode policies extracted from the optimal mix. Note for Germany the relationship between the dollar value and preferred mode. The higher the dollar value the more sea mode is preferred. For Vietnam the same relationship is there and also the sea mode is preferred more often than the air mode.

This pattern is expected since the intra-theatre shipping times used are the same for each mode. The shipping times used from CONUS to theater depot are shown in the following table. These times were obtained from RAC.

TABLE 19-A

Theater	Mode	Time Days
Germany	Sea	40
Germany	Air	9
Vietnam	Sea	49
Vietnam	Air	9

4.4 Transport Mode

Normally the transport decision and the stockage decision are made sequentially. Once a particular transport mode is selected, the required stockage for that mode is determined. But a tradeoff exists between transportation cost and inventory cost. Typically, inventory models require as input the resupply time along with the item characteristics to develop stockage policies. The transport mode, of course, determines what the resupply time will be. Since the number of feasible transport modes from the supplier to the stockage point is usually small, it is a simple matter to apply the inventory model to each transport mode and make a total cost comparison of each.

This is precisely what was done using the multi-echelon model and RAC's transportation cost algorithm. Table 20 presents a summary of the results concerning the transport modes. The costs presented are for the entire 710 item catalog and are in a per year basis.

TABLE 20

TRANSPORTATION COST VS TOTAL COST

	Total Cost	Tran Cost	Trans-% of Total
Germany Sea	$ 20317	5613	27.6
Germany Air	22229	11636	52.3
Vietnam Sea	38163	9959	26.1
Vietnam Air	53011	33177	62.6

Transportation costs are a sizeable proportion of the total costs in all 4 cases. The air mode being more expensive results in a higher percentage of total cost than does the sea mode. However, the reduction in inventory required under the air mode is not sufficient to overcome the increased transport cost. On a dollar basis, the sea mode is preferred for both the German and Vietnam theaters. Use of the sea mode results in a savings of about $2000 per year in Germany and $15000 per year in Vietnam.

Table 21 presents an interesting result concerning the optimal mixing of transport modes by item. These results are for percent of failures detected at GSU $\neq 0$. Of the 710 items, only 190 (269) preferred the Germany (Vietnam) sea mode while 520 (441) preferred air.

TABLE 21

Mode	# Items Preferring Mode	Opt Cost	Opt Tran Cost
Germany Sea	190	18403	6110
Germany Air	520		
Vietnam Sea	269	33714	10031
Vietnam Air	441		

Seemingly, this would indicate the preference of an all air mode over an all sea policy. However, a strong relationship exists between this item's failure rate and its preferred transport mode. Almost without exception, the very low failure rate items prefer the air mode. These are the items

which are most abundant but which account for only a small portion of total cost. If an all air policy were imposed on all of the items, therefore, a severe penalty would be incurred by the items with the higher failure rates. On the other hand, if an all sea policy is imposed, the penalties incurred by the low failure rate items would be mild in comparison.

TABLE 22

	Germany Sea	Germany Air
Vietnam Sea	180 (25.35%)	89 (12.46%)
Vietnam Air	10 (1.41%)	431 (60.78%)

Table 22 indicates the relationship between preferred modes by theater. For example 180 or (25.35%) of the items prefer the sea mode for both theaters, while 431 (60.78%) prefer the air mode to both theaters. Only about 14% of the items prefer different modes by theater.

The primary difference between the theaters is their distrance from CONUS. The European theater is approximately 4500 miles away, and Vietnam theater is 8410 miles away. Notice that when a mix mode by theater is preferred it is usually the Germany Air, Vietnam Sea mix. Thus, some items which preferred the Vietnam sea mode rejected the Germany Sea mode in favor of Germany Air. Sea transport costs do not depend as strongly on distance as do air costs. At shorter distances the air mode becomes preferable. It is not surprising the predominant shift to mix modes should favor Germany Air, Vietnam Sea. Nevertheless, the shift is not drastic. In fact, a strong tendency towards the same mode is very evident. Item characteristics appear to be more important to the selection of the transport mode than do the theater characteristics.

4.5 Military Essentiality Rankings

A report published by this office entitled "Military Essentiality Coding" presents the results of a project which developed the military essentiality codes for the parts of the rough terrain forklift. The concept of military essentiality employed in that project was that "the military essentiality of a repair part is areflection of the degree to which its failure, if and when it occurs, affects the ability of the seapons system to perform its intended mission". The military essentiality codes are a "systematic way of expressing the expected degree of degradation in weapon system performance when the part fails." A ranking scheme was developed which placed the part in a particular class depending upon the answers obtained from questionnaires. In the Army system today, items classified critical are stocked even though the stockage may not be demand supported. Along with more sophisticated military essentiality ranking schemes should also come more sophisticated

stockage decision processes employing the essentiality rankings. This section explains an attempt to use the multi-echelon model along with the essentiality rankings to develop optimum stockage policies.

The concept of backorder cost frequently employed in stockage decisions is closely related to the concept of military essentiality. A backorder cost should reflect the loss suffered in the mission because of the backorder. Since military essentiality of a part reflects the degree to which the weapons system can perform its mission upon failure of the part, the military essentiality should be used to reflect the degree of the penalty cost applied to a backorder. Because military essentiality is still only a concept and not a precisely defined property, we are free to introduce military essentiality measure in the multi-echelon model as we please so long as it conforms with the concept of military essentiality. In the cost equation presented earlier, therefore, the backorder cost, C_b was weighted by an essentiality measure E. Accordingly, the measure E must be developed dependent on its use in the model. But, the same reason which prevents the measurement of backorder costs also hinders the measurement of essentiality; mission effectiveness itself has not been suitably defined to allow the definition of concepts related to it. The military essentiality project, however, went no further than developing a ranking measure for essentiality. To use essentiality, though, a scheme for developing a cardinal measure must be found.

After a limited investigation, the only realistic approach uncovered was to set target availabilities for each essentiality class and determine the E values which yield these availabilities.

This approach seems appealing because it allows a relationship between logistics performance and essentiality. Nevertheless, there are several drawbacks. For one, the relationship between availability and essentiality is established a priori by a decision maker on a purely subjective basis. Ideally, this relationship should exist only a posteriori since essentiality is meant to be one of the input factors to the decision process, and logistics performance the output. Another and equally serious fault of this approach is that the price of the item is eliminated from the decision process. Items which have identical characteristics except for price are, therefore, stocked in the same amount.

On the other hand, this approach, since it is a systematic procedure, would improve upon the decisions as made today. Currently, items are classified only critical or not critical by the Army. If a critical item does not meet the demand support requirements one is stocked away. But if it does meet the requirements, there is no difference in its stockage level than the stockage levels as computed for non-critical item of the same characteristics. The availability approach presented above is a definite improvement over the current procedures. Moreover, a partial dollar saving is introduced through the varying availability levels. Thus, while complete satisfaction cannot be reported with the approach, it is, nevertheless, a very definite improvement over the current methods.

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13. ABSTRACT

An S,S-1 multi-echelon model is used to develop optimal stockage policies for a catalog of items. The items were the repair parts of a rough terrain forklift on which a military essentiality methodology had been tested. The model included such factors as holding cost, performance objective, military essentiality, and transportation cost. An optimization algorithm was developed and a constraint manipulation technique experimented with.

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